Nearly collision trajectories of the 3 body problem

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We consider plane 3 body problem with one mass of order 1 and two other of order \( \mu \ll 1 \). In the first approximation in \( \mu \), small bodies move along nearly Kepler orbits. When they undergo almost collisions, they interact strongly, and start moving along a new set of nearly Kepler orbits. This singularly perturbed dynamical system was first considered by Poincaré. It turns out that if the dynamical system is replaced by a variational problem, then the perturbation becomes regular, and the limit system is a billiard type problem with elastic collisions of small bodies. We show that it is possible to shadow non degenerate periodic and chaotic billiard orbits by trajectories of the 3 body problem with error of order \( \mu | \ln \mu | \). The proof of the main result is based on the Levi–Civita regularization and a generalization of Shilnikov Lemma to Hamiltonian systems possessing a nondegenerate normally hyperbolic symplectic critical manifold.

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