

Stability of a Hamiltonian system in a limiting case

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We give a fairly simple geometric proof that an equilibrium point of a Hamiltonian system of two degrees of freedom is Liapunov stable in a degenerate case. That is the $1: -1$ resonance case where the linearized system has double pure imaginary eigenvalues $\pm i\omega$, $\omega \neq 0$ and the Hamiltonian is indefinite. The linear system is weakly unstable, but if a particular coefficient in the normalized Hamiltonian is of the correct sign then Moser’s invariant curve theorem can be applied to show that the equilibrium point is encased in invariant tori and thus it is stable.

This result implies the stability of the Lagrange equilateral triangle libration point, \mathcal{L}_4 , in the planar circular restricted three-body problem when the mass ratio parameter is equal to μ_R , the critical value of Routh.

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