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A numerical splitting scheme for very degenerate advection-diffusion-reaction equations

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In this talk I will describe some *ongoing work with T.O. Gallouët and M. Laborde* on the numerical approximation of very degenerate advection-diffusion-reaction equations of the form $\partial_t \rho = \mathrm{div}(\rho\nabla(p + V(x)) + \rho\Phi(p, x)$, where the potential drift $V(x)$ is prescribed and the heterogeneous reaction term $\Phi(., x)$ is given. The diffusion nonlinearity can be of the Porous-Medium type $p = g_m(\rho) = \frac{m}{m-1}\rho^{m-1}$, and in the limit $m \to \infty$ the diffusion degenerates into a Hele-Shaw type problem with hard constraint $\rho \leq 1$ and the maximal monotone graph relation: $p \in g_\infty(\rho) = \{0\}$ if $0 \leq \rho < 1$, and $p \in g_\infty(\rho) = [0, \infty)$ if $\rho = 1$. Based on the Kantorovich-Fisher-Rao distance, recently introduced for nonnegative measures, we derive a semi-discrete splitting scheme similar to the classical JKO one, and prove convergence towards weak solutions as the time step goes to zero. The corresponding fully discrete scheme can be easily implemented using any existing Monge-Ampère solvers, and provides a good approximation and satisfactory results in agreement with the qualitative behaviour of solutions.

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