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Kernels of vector-valued Toeplitz operators

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Let S be the shift operator on the Hardy space H^2 and let S^* be its adjoint. A closed subspace $\mathcal{F}$ of H^2 is said to be nearly S^*-invariant if every element $f \in \mathcal{F}$ with $f(0) = 0$ satisfies $S^* f \in \mathcal{F}$. In particular, the kernels of Toeplitz operators are nearly S^*-invariant subspaces. Hitt gave the description of these subspaces. They are of the form $\mathcal{F} = g(H^2 \ominus uH^2)$ with $g \in H^2$ and u inner, $u(0) = 0$. A very particular fact is that the operator of multiplication by g acts as an isometry on $H^2 \ominus uH^2$. Sarason obtained a characterization of the functions g which act isometrically on $H^2 \ominus uH^2$. Hayashi obtained the link between the symbol φ of a Toeplitz operator ($T_\varphi f = p_+(\varphi f)$) and the functions g and u to ensure that a given subspace $\mathcal{F} = gK_u$ is the kernel of T_φ. Chalendar, Chevrot and Partington studied the nearly S^*-invariant subspaces for vector-valued functions. In this talk, we investigate the generalization of Sarason's and Hayashi's results in the vector-valued context.

The talk is in two parts. The first one, I present the scalar results of Hitt, Sarason and Hayashi and the ingredients needed. In the second part treats the vectorial case. I explain which difficulties are specific. I conclude with the entire description of kernels of Toeplitz operators with a matricial symbol.