

A Celebration of Raoul Botts Legacy in Mathematics
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Dirac operators and K -theory for discrete groups

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Abstract

Let G be a locally compact, Hausdorff, and second countable (i.e. the topology has a countable base) topological group. Examples are Lie groups, p -adic groups, adelic groups, and discrete groups. In 1980, P. Baum and A. Connes proposed an answer to the problem of calculating the K -theory of the reduced C^* algebra of G . When true, the conjecture has several corollaries. Among these are exhaustion of the discrete series by Dirac induction, Novikov conjecture on homotopy invariance of higher signatures, and Gromov–Lawson–Rosenberg conjecture on Riemannian metrics of positive scalar curvature. Since 1980, the conjecture has been proved for many examples. Although it seems quite possible that eventually there will be a discrete group which is a counter-example, at the present moment there is no really good candidate for a counter-example. This talk will explain the conjecture, concentrating on the case of discrete groups from the point of view of the index of elliptic operators. The talk is intended for non-specialists. All the basic definitions (C^* algebra, K -theory etc) will be carefully stated.