

Totally Silver Graphs

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Silver matrix:

1	2	3	4	5	6
7	1	5	3	6	4
8	10	1	6	4	2
9	8	11	1	2	5
10	11	9	7	1	3
11	9	7	10	8	1

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Theorem (Mahdian, Mahmoodian; 2000)

A silver matrix of order n exists, if and only if $n = 1$ or $2|n$.

Silver Cubes

					5	1	4	
			6	7	1	1	6	7
1	2	3	3	1	2	2	3	1
4	5	1	1	4	5			
7	1	6						

Silver Cubes

					5	1	4	
			6	7	1			
			3	1	2	1	6	7
			1	4	5	2	3	1
1	2	3						
4	5	1						
7	1	6						

Theorem (G, Goddyn, Mahmoodian, Verdian; 2008)

A silver cube of order n exists, if $n = 2^a 3^b 5^c$.

Definition

An (n, d) -*silver cube* is any

$$c : V(K_n^d) \rightarrow \{0, 1, \dots, d(n-1)\}$$

in which every vertex of a *diagonal* is *rainbow*.

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- $d = 3$: silver cubes

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Open Problems

- K_{11}^3 (K_p^3 is silver where $p \geq 11$ is a prime)

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- K_{11}^3 (K_p^3 is silver where $p \geq 11$ is a prime)
- Q_{20} (Q_n is not silver where $4|n$ and $n \neq 2^t$)

Definition

Given an r -regular G and a diagonal I ,

$$c : V(G) \rightarrow \{0, 1, \dots, r\}$$

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$$c : V(G) \rightarrow \{0, 1, \dots, r\}$$

is a *silver colouring* if every $v \in I$ is rainbow. *Totally silver* if every $v \in V(G)$ is rainbow.

G is said to be *(totally) silver*, if it admits a (totally) silver colouring.

Observation

G is totally silver iff G is *domatically full*, i.e. $V(G)$ admits $\delta(G) + 1$ disjoint dominating sets.

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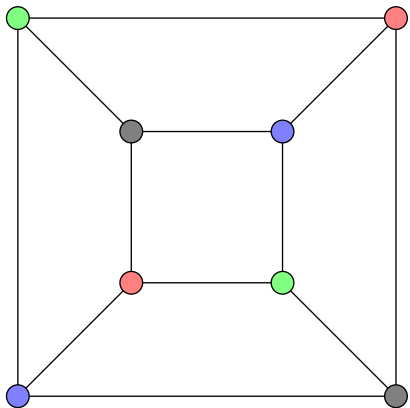
Observation

G is totally silver iff $\chi(G^2) = r + 1$ where G^2 is the *square* of G .

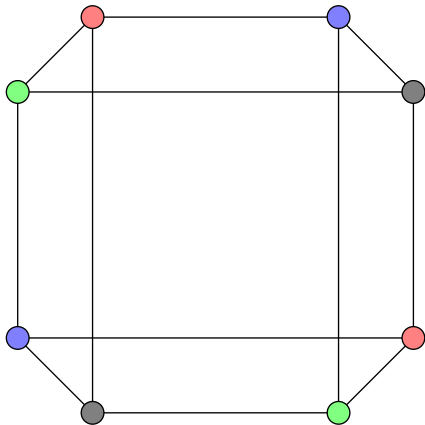
Examples of Totally Silver Graphs

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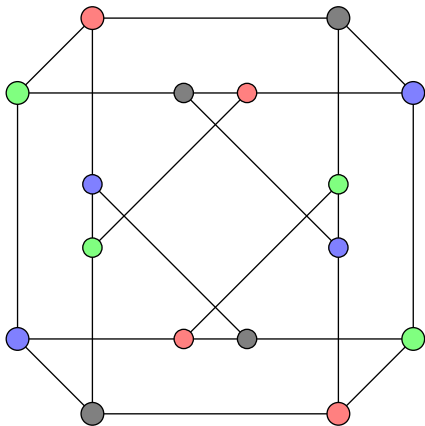
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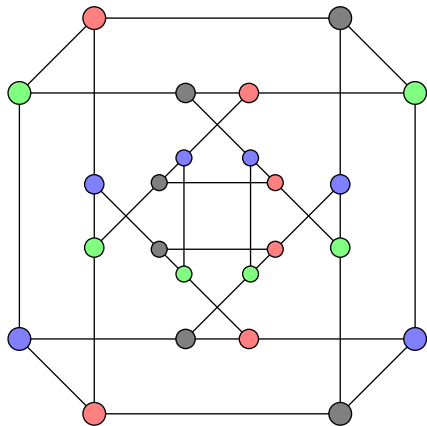
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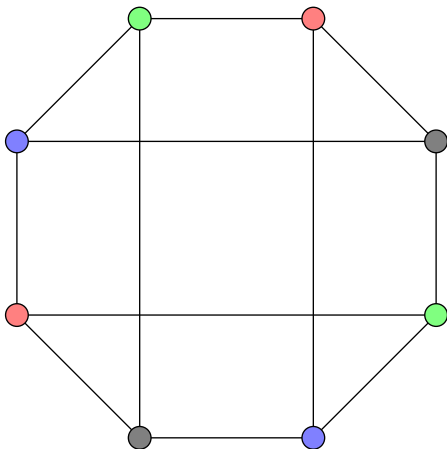
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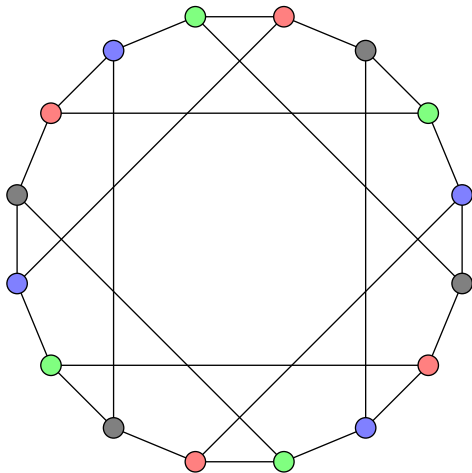
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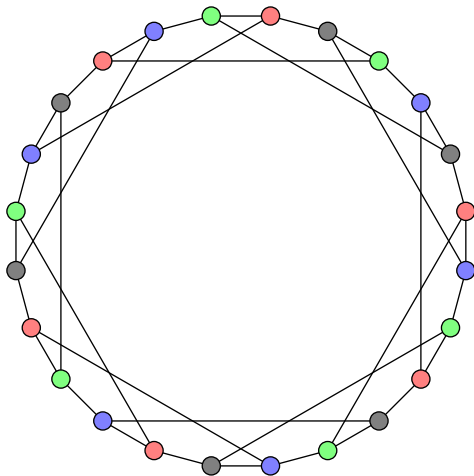
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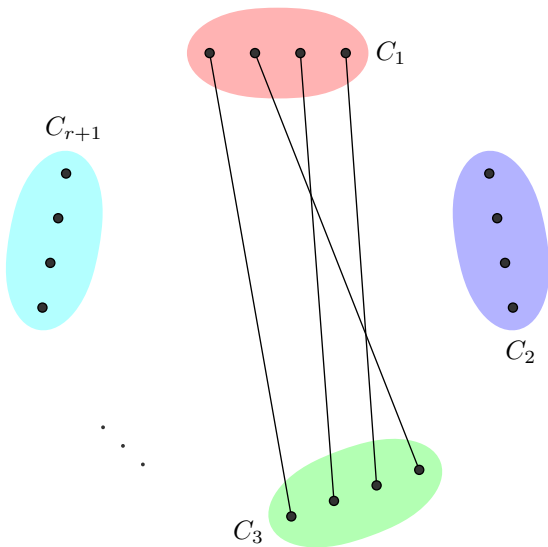
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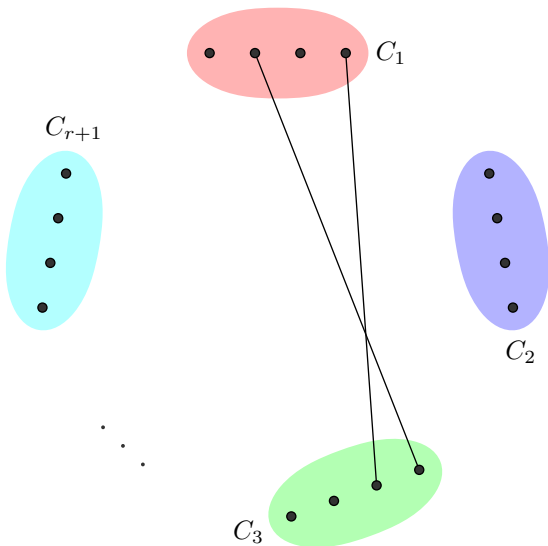
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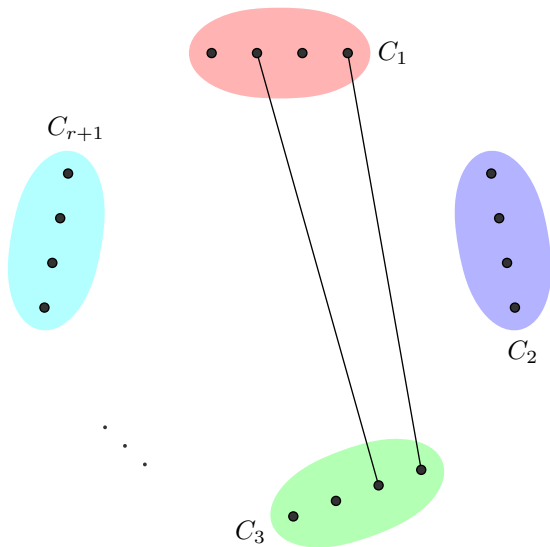
Characterization of Totally Silver Graphs



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Theorem

Every r -regular totally silver graph can be obtained by a sequence of (coloured) 2-switches from a disjoint union of copies of $r + 1$ -cliques.

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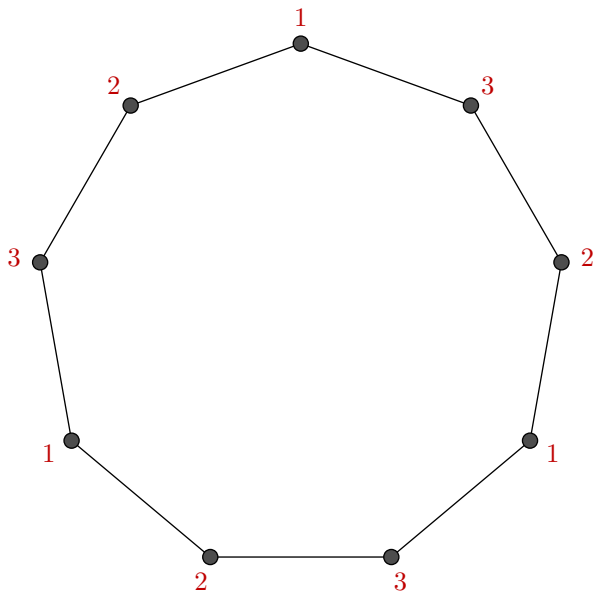
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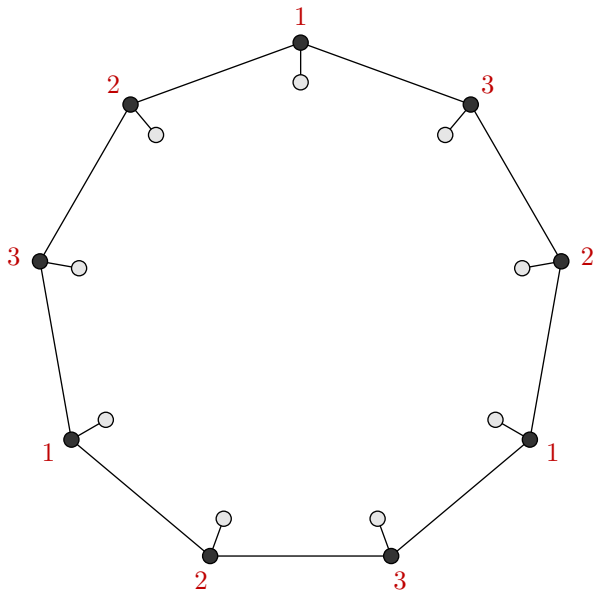
Question

Do there exist totally silver cubic graphs of high girth?

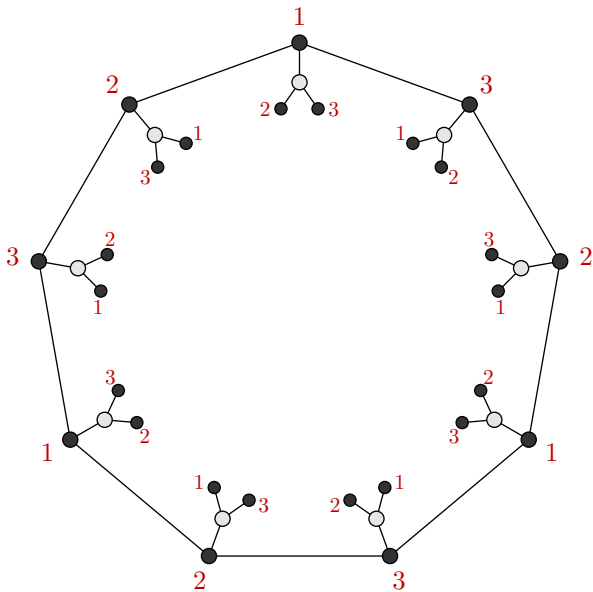
An Example with Girth 9



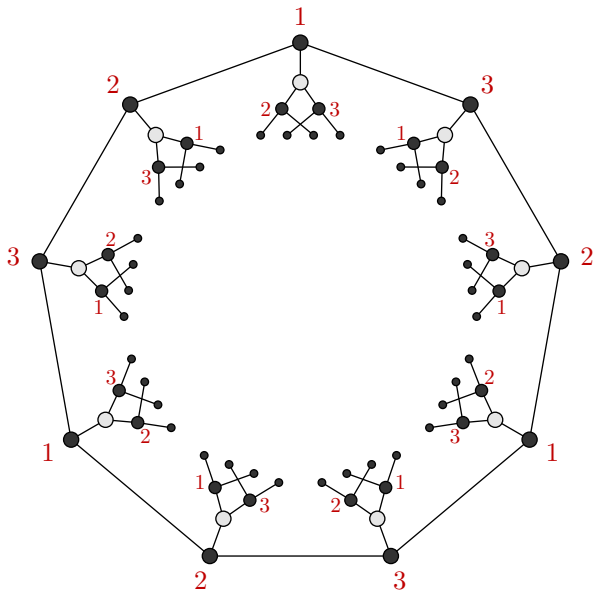
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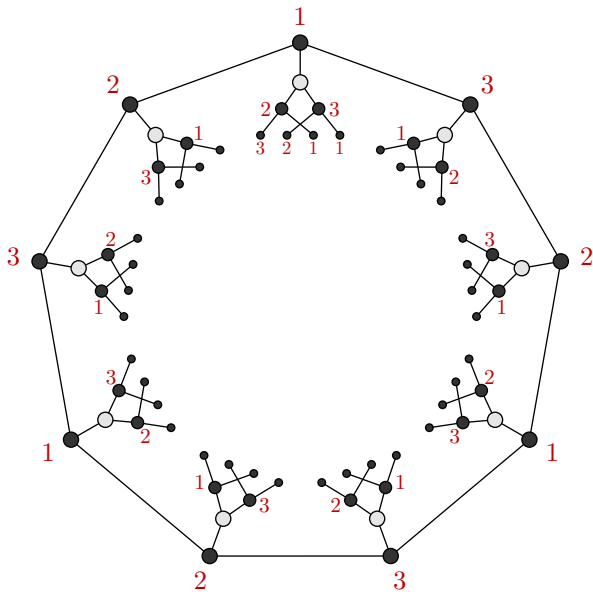
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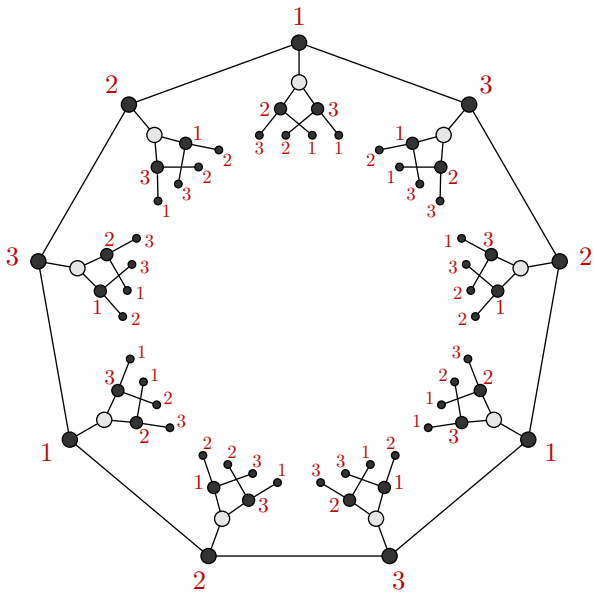
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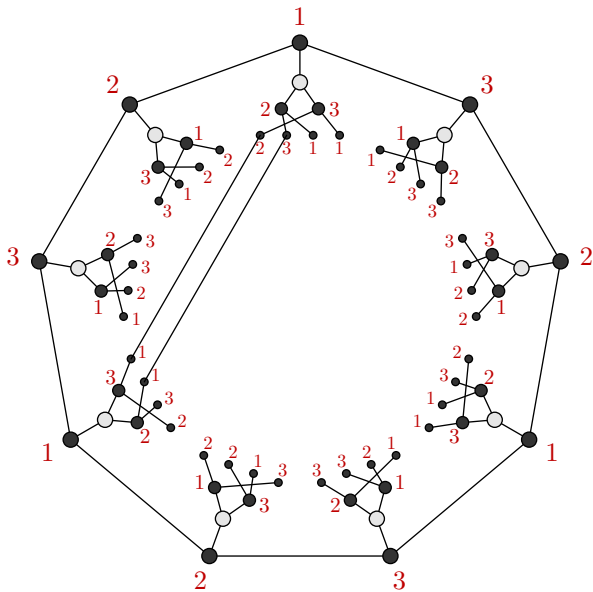
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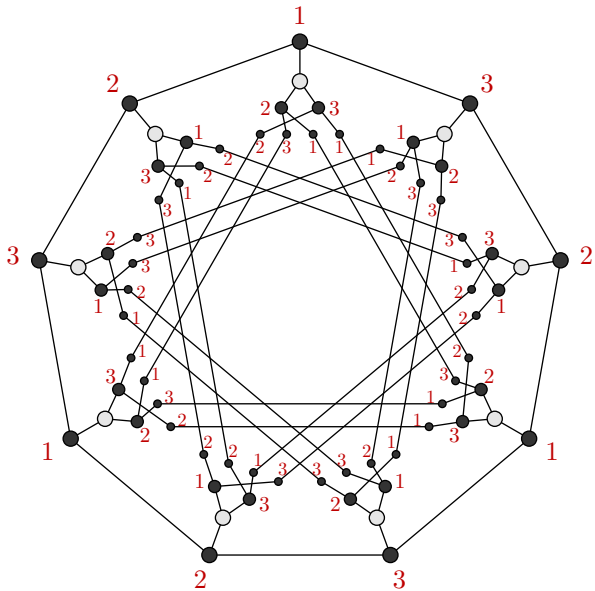
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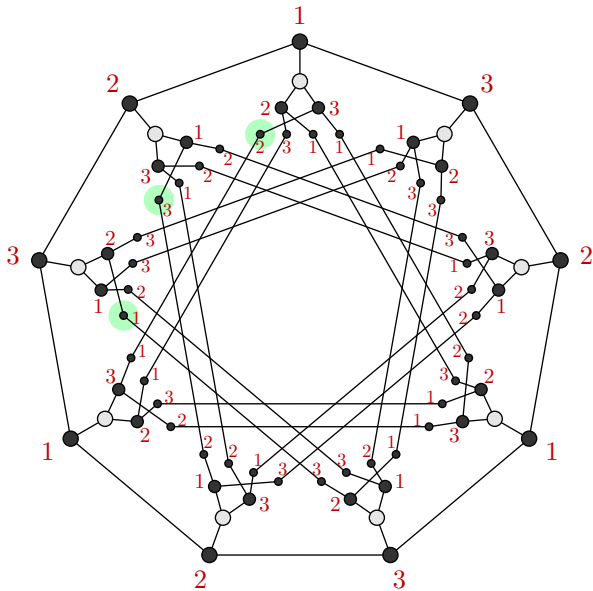
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Thank you!