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A kinematic formula for Gaussian excursions

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Kinematic formulae are results which give concise expressions for the mean values of geometric characteristics of random sets.

In the setting of this talk, the random sets will be Gaussian excursion sets of the form $f^{-1}(D)$, where f is a smooth, vector valued Gaussian random process on a general parameter space, typically a stratified manifold. Two simple examples might be provided by f taken to be a real valued planar random wave on a smooth domain and D the positive real line or the set containing only the origin. In the former case the excursion set is no more than the nodal domain of f, while in the latter it is its boundary.

The corresponding geometric characteristics will be those of integral geometry, including volume, surface area, Euler characteristic and various measures of cross-sectional size. As families, these characteristics go by names such as Minkowski functionals and Lipshitz–Killing curvatures.

Building on a fundamental result of Jonathan Taylor, I shall describe a general, concise and explicit formula for the mean value of these characteristics of Gaussian excursion sets, along with some of its implications and applications and a brief description of a (rather long) proof.

This is joint work with Jonathan Taylor.