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## Dimers and Harnack curves

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A polynomial  $P(z, w)$  with real coefficients is said to be Harnack if the real components of  $P(z, w) = 0$  satisfy a certain simple geometric property. These polynomials are somewhat analogous to one-variable polynomials with only real, negative roots. They also arise in the partition function of the planar dimer model, where they play the role of “spectral curve” of the underlying Kasteleyn matrix. Remarkably, every Harnack curve arises from a dimer model, and dimer models thus provide a convenient parameterization of the space of Harnack curves. Conversely, the fact that Harnack curves have only “nice” singularities means that the dimer models have well-understood phase transitions.

**This is joint work with Andrei Okounkov.**